

Image Denoising Using Hybrid Graph Laplacian Regularization

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Abstract— In real world application, recovery of image is necessary. We present a method for removing noise from digital images corrupted with additive, multiplicative, and mixed noise. In this paper, we propose a unified framework to perform progressive image recovery based on hybrid graph Laplacian regularized regression. We first construct a multiscale representation of the target image by Laplacian pyramid, then progressively recover the degraded image in the scale space from coarse to fine so that the sharp edges and texture can be eventually recovered. Also, a graph Laplacian regularization model represented by implicit kernel is learned, which simultaneously minimizes the least square error on the measured samples and preserves the geometrical structure of the image data space. In this procedure, the intrinsic manifold structure is explicitly considered using both measured and unmeasured samples, and the nonlocal self-similarity property is utilized as a fruitful resource for abstracting a priori knowledge of the images. On the other hand, between two successive scales, the proposed model is extended to a projected high-dimensional feature space through explicit kernel mapping to describe the interscale correlation, in which the local structure regularity is learned and propagated from coarser to finer scales. In this way, the proposed algorithm gradually recovers more and more image details and edges, which could not be recovered in previous scale

Index Terms— Image denoising, graph Laplacian, kernel theory, local smoothness.

I. INTRODUCTION

The recovery of image from corrupted observation is the problem that encouraged in many engineering and science applications, consumer electronics to medical imaging. In many practically observed images contains noise that should be removed beforehand for improving the virtual pleasure and the reliability of subsequent image analysis task. Image contains the various types of noise and the impulse noise is one of the most frequently happened noise, which may be introduced into image during acquisition and transmission. Hence, In this paper we focus on the impulse noise removal.

It's challenging image processing problem to remove impulse noise from image, because edges which can also be modeled as abrupt intensity jumps in a scan line are highly salient features for visual attention. Therefore, besides impulse noise removal, another important requirement for image denoising procedures is that they should preserve important image structures, such as edges and major texture features.

Image denoising is an important image processing task, both as a process itself, and as a component in other processes. Very many ways to denoise an image or a set of data exists. The main properties of a good image denoising model is that it will remove noise while preserving edges. Traditionally, linear models have been used. One common approach is to use a Gaussian filter, or equivalently solving the heat-equation with the noisy image as input-data, i.e. a linear, 2nd order PDE-model. For some purposes this kind

of denoising is adequate. One big advantage of linear noise removal models is the speed. But a drawback of the linear

models is that they are not able to preserve edges in a good manner: edges, which are recognized as discontinuities in the image, are smeared out. Nonlinear models on the other hand can handle edges in a much better way than linear models can. One popular model for nonlinear image denoising is the Total Variation (TV)-filter, introduced by Rudin, Osher and Fatemi. This filter is very good at preserving edges, but smoothly varying regions in the input image are transformed into piecewise constant regions in the output image. Using the TV-filter as a denoiser leads to solving a 2nd order nonlinear PDE. Since smooth regions are transformed into piecewise constant regions when using the TV-filter, it is desirable to create a model for which smoothly varying regions are transformed into smoothly varying regions, and yet the edges are preserved.

II. LITERATURE SURVAY

There are variety of impulse noise removal methods. In general the results of the noise removal have a strong influence on the quality of the image processing technique. Several techniques for noise removal are well established in image processing. The nature of the noise removal problem depends on the type of the noise corrupting the image. In the field of image noise reduction several linear and non linear filtering methods

have been proposed. Linear filters are not able to effectively eliminate impulse noise as they have a tendency to blur the edges of an image. On the other hand non linear filters are suited for dealing with impulse noise. Several non linear filters based on Classical and fuzzy techniques have emerged in the past few years.

Noise is the result of errors in the image acquisition process that results in pixel values that do not reflect the true intensities of the real scene. Noise reduction is the process of removing noise from a signal. Noise reduction techniques are conceptually very similar regardless of the signal being processed, however a priori knowledge of the characteristics of an expected signal can mean the implementations of these techniques vary greatly depending on the type of signal. The image captured by the sensor undergoes filtering by different smoothing filters and the resultant images. All recording devices, both analogue and digital, have traits which make them susceptible to noise. The fundamental problem of image processing is to reduce noise from a digital color image. The two most commonly occurring types of noise are i) Impulse noise, ii) Additive noise (e.g. Gaussian noise) and iii) Multiplicative noise (e.g. Speckle noise).

Noise is removed from images by filtering. impulse noise removal poses a fundamental challenge for conventional linear methods. They typically achieve the target of noise removal by low-pass filtering which is performed by removing the high-frequency components of images. This is effective for smooth regions in images. But for texture and detail regions, the low-pass filtering typically introduces large, spurious oscillations near the edge known as Gibb's phenomena. Accordingly, nonlinear filtering techniques are invoked to achieve effective performance. One kind of the most popular and robust nonlinear filters is the so called decision-based filters, which first employ an impulse noise detector to determine which pixels should be filtered and then replace them by using the median filter or its variants, while leaving all other pixels unchanged. The representative methods include the adaptive median filter (AMF) and the adaptive center-weighted median filter (ACWMF).

III. EXISTING METHOD

There are various successful frameworks for impulse noise removal that are derived from the energy methods. In this framework, image denoising is considered as a variational problem where a restored image is computed by a minimization of some energy functions. Typically, such functions consist of a fidelity term such as the norm difference between the recovered image and the noisy image, and a regularization term which penalizes high frequency noise. In this the algorithm are used for deblurring and denoising and achieve wonderful objective and subjective performance.

From a statistical perspective, recovering images from degraded forms is inherently an ill-posed inverse problem. It

often can be formulated as an energy minimization problem in which either the optimal or most probable configuration is the goal. The performance of an image recovery algorithm largely depends on how well it can employ regularization conditions or priors when numerically solving the problem, because the useful prior statistical knowledge can regulate estimated pixels. Therefore, image modeling lies at the core of image denoising problems.



Fig.1. Intra-scale and inter-scale correlation.

One common prior assumption for natural images is *intensity consistency*, which means: (1) nearby pixels are likely to have the same or similar intensity values; and (2) pixels on the same structure are likely to have the same or similar intensity values. Note that the first assumption means images are locally smooth, and the second assumption means images have the property of non-local self-similarity. Accordingly, how to choose statistical models that thoroughly explore such two prior knowledge directly determines the performance of image recovery algorithms. Another important characteristic of natural images is that they are comprised of structures at different scales. Through multi-scale decomposition, the structures of images at different scales become better exposed, and hence be more easily predicted.

Early heuristic observation about the local smoothness of image intensity field has been quantified by several linear parametric models, such as the piecewise autoregressive (PAR) image model. Moreover, the study of natural image statistics reveals that the second order statistics of natural images tends to be invariant across different scales, as illustrated in Fig. 1 (denoted by inter-scale correlation). And those scale invariant features are shown to be crucial for human visual perception. This observation inspires us to learn and propagate the statistical features across different scales to keep the local smoothness of images. On the other hand, the idea of exploiting the non-local self-similarity of images has attracted increasingly more attention in the field of image processing. Referring to Fig. 1 (denoted by intra-scale correlation), the non-local self-similarity is based on the observation that image patches tend to repeat themselves in the whole image plane, which in fact reflects the intra-scale correlation. All those findings tell us that local non-local redundancy and intra-inter-scale correlation can be thought of as two sides of the same coin. The multiscale framework provides us a wonderful choice to efficiently

combine the principle of local smoothness and non-local similarity for image recovery.

Moreover, recent progress in semi-supervised learning gives us additional inspiration to address the problem of image recovery. Semi-supervised learning is motivated by a considerable interest in the problem of learning from both labeled (measured) and unlabeled (unmeasured) points. Specially, geometry-based semi-supervised learning methods show that natural images cannot possibly fill up the ambient Euclidean space rather it may reside on or close to an underlying submanifold. In this paper, we try to extract this kind of low dimensional structure and use it as prior knowledge to regularize the process of image denoising. In another word, in the algorithm design, we will explicitly take into account the intrinsic manifold structure by making use of both labeled and unlabeled data points. Motivated by the above observation, the well-known theory of kernels and works on graph-based signal processing, in this paper, we propose a powerful algorithm to perform progressive image recovery based on hybrid graph Laplacian regularized regression. In our method, a multi-scale representation of the target image is constructed by Laplacian pyramid, through which we try to effectively combine local smoothness and non-local self-similarity. On one hand, within each scale, a graph Laplacian regularization model represented by implicit kernel is learned which simultaneously minimizes the least square error on the measured samples and preserves the geometrical structure of the image data space by exploring non-local self-similarity. In this procedure, the intrinsic manifold structure is considered by using both measured and unmeasured samples. On the other hand, between two scales, the proposed model is extended to the parametric manner through explicit kernel mapping to model the interscale correlation, in which the local structure regularity is learned and propagated from coarser to finer scales.

IV. PROPOSED SYSTEM

The proposed model is extended to the parametric manner through explicit kernel mapping to model the interscale correlation, in which the local structure regularity is learned and propagated from coarser to finer scales. It is worth noting that the proposed method is a general framework to address the problem of image recovery. We choose one typical image recovery task, impulse noise removal, but not limit to this task, to validate the performance of the proposed algorithm. Moreover, in our method the objective functions are formulated in the same form for intra-scale and inter-scale processing, but with different solutions obtained in different feature spaces: the solution in the original feature space by implicit kernel is used for intra-scale prediction, and the other solution in a higher feature space mapped by explicit kernel is used for inter-scale prediction. Therefore, the proposed image recovery algorithm actually casts the consistency of local and global

correlation through the multi-scale scheme into a unified framework.

The proposed method we introduced graph Laplacian regularized model and its kernel-based optimization solutions, multi-scale image recovery framework.

V. IMAGE RECOVERY VIA GRAPH LAPLACIAN REGULARIZED REGRESSION

Problem description

Given a degraded image X with n pixels, each pixel can be described by its feature vector $x_i = [u_i, b_i] \in \mathbb{R}^{m+2}$, where $u_i = (h, w)$ is the coordinate and $b_i \in \mathbb{R}^m$ is a certain context of x_i which is defined differently for different tasks. All pixels in the image construct the sample set $\chi = \{x_1, x_2, \dots, x_n\}$. We call the grayscale value y_i as the label of x_i . For image impulse noise removal, when image measures are noise-dominated, the performance of image recovery can be improved by implementing it in two steps: The first step is to classify noisy and clean samples by using the adaptive median filter or its variant, which depends on the noise type. Then, noisy samples are treated as unlabeled ones with their intensity values to be re-estimated, and the rest clean samples are treated as labeled ones with intensity values unchanged. The second step is to adjust the inference to give a best fit to labeled measures and uses the fitted model to estimate the unlabeled samples. In view of machine learning, this task can be addressed as a problem of semi-supervised regression.

Graph Laplacian Regularized Regression (GLRR)

What we want to derive is the prediction function f , which gives the re-estimated values of noisy samples. Given labeled samples $X_l = \{(x_1, y_1), \dots, (x_l, y_l)\}$ as the training data, one direct approach of learning the prediction function f is to minimize the prediction error on the set of labeled samples, which is formulated as follows:

$$\arg \min_{f \in H_\kappa} J(f) = \arg \min_{f \in H_\kappa} \sum_{i=1}^l \|y_i - f(x_i)\|^2 + \lambda \|f\|^2, \quad (1)$$

where H_κ is the Reproducing Kernel Hilbert Space (RKHS) associated with the kernel κ . H_κ will be the completion of the linear span given by $\kappa(x_i, \cdot)$ for all $x_i \in \chi$, i.e., $H_\kappa = \text{span}\{\kappa(x_i, \cdot) | x_i \in \chi\}$.

The above regression model only makes use of the labeled samples to carry out inference. When the noise level is heavy, which means there are few labeled samples, it is hard to achieve a robust recovery of noisy image. Moreover, it fails to take into account the intrinsic geometrical structure of the image data. Note that we also have a bunch of unlabeled samples $\{x_{l+1}, \dots, x_n\}$ at hand. In the field of machine learning, the success of semi-supervised learning is plausibly due to effective utilization of the large amounts of unlabeled data to extract information that is useful for generalization. Therefore, it is reasonable to leverage both labeled and unlabeled data to achieve better predictions.

In order to make use of unlabeled data, we follow the well-known manifold assumption, which is implemented by a graph structure. Specially, the whole image sample set is

modeled as a undirected graph, in which the vertices are all the data points and the edges represent the relationships between vertices. Each edge is assigned a weight to reflect the similarity between the connected vertices. As stated by the manifold assumption, data points in the graph with larger affinity weights should have similar values. Meanwhile, with the above definition, the intrinsic geometrical structure of the data space can be described by the graph Laplacian. Through the graph Laplacian regularization, the manifold structure can be incorporated in the objective function. Mathematically, the manifold assumption can be implemented by minimizing the following term:

$$R(f) = \frac{1}{2} \sum_{i,j} (f(x_i) - f(x_j))^2 W_{ij} \quad (2)$$

where W_{ij} is in inverse proportion to $d_2(x_i, x_j)$. W_{ij} is defined as the edge weight in the data adjacency graph which reflects the affinity between two vertices x_i and x_j . In graph construction, edge weights play a crucial role. In this paper, we combine the edge-preserving property of bilateral filter [16] and the robust property of non-local-means weight [15] to design the edge weights, which are defined as follows:

$$W_{ij} = \frac{1}{C} \exp \left\{ -\frac{\|u_j - u_i\|^2}{\sigma^2} \right\} \exp \left\{ -\frac{\|b_j - b_i\|^2}{\varepsilon^2} \right\}, \quad (3)$$

$\sigma > 0, \varepsilon > 0.$

where b_i (b_j) is defined as the local patch centered on u_i (u_j). The first exponential term considers the geometrical nearby, and the second one considers the structural similarity.

Let D be a diagonal matrix, whose diagonal elements are the row sums of W , i.e., $D(i, i) = \sum_j W_{ij}$. We define $L = D - W \in \mathbb{R}^{n \times n}$ as the graph Laplacian. With above definition, Eq.(2) can be further written as:

$$R(f) = \sum_{i=1}^n f(x_i)^2 \sum_{j=1}^n W_{ij} - \sum_{i,j} W_{ij} f(x_i) f(x_j) = f^T D f - f^T W f = f^T L f, \quad (4)$$

where $f = [f(x_1), \dots, f(x_n)]$. Combining this regularization term with Eq.(1), we obtain the objective function of Laplacian regularized least square (LapRLS)

$$\arg \min_{f \in \mathcal{H}_k} \{ J(f) = \|y_L - f_L\|^2 + \lambda \|f\|^2 + \gamma f^T L f \}, \quad (5)$$

where $y_L = [y_1, \dots, y_L]^T$, $f_L = [f(x_1), \dots, f(x_l)]^T$ and $f = [f(x_1), \dots, f(x_n)]^T$.

C. Optimization by Implicit Kernel

In order to obtain the optimal solution for the above objective function, we exploit a useful property of RKHS, the so called representer theorem. It states that minimizing

of any optimization task in Hilbert space H has finite representation in H .

VI. PROGRESSIVE HYBRID GRAPH LAPLACIAN REGULARIZATION

In this paper, we propose to use a simple multi-scale framework to achieve such a purpose. There are at least several reasons why we use the multi-scale framework. First, one important characteristic of natural images is that they are comprised of structures at different scales. Through multi-scale decomposition, the structures of images at different scales become better exposed, and hence be more easily predicted. Second, a multi-scale scheme will give a more compact representation of imagery data because it encodes low frequency parts and high frequency parts separately. As well known, the second order statistics of natural images tends to be invariant across different scales. Therefore, the low frequency parts can be extracted from much smaller

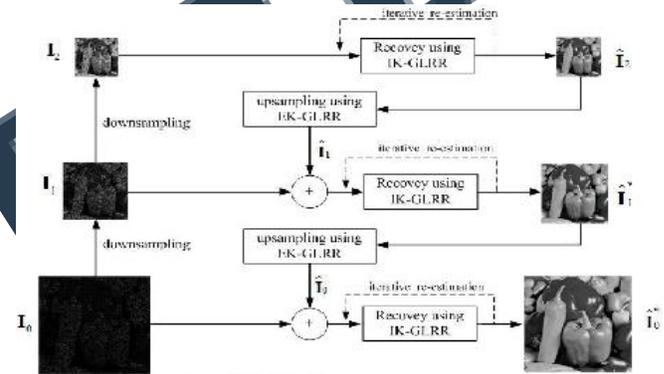


Fig. 2 Diagram of the proposed method downsampling image. Third, the stronger correlations among adjacent image blocks will be captured in the downsampled images because every four image blocks are merged into one block in the downsampled image. As a consequence, in this paper, we propose an effective approach to recover noisy imagery data by combining hybrid models and the multi-scale paradigm.

We now introduce the multiscale implementation of the proposed method. To give a clear illustration, we summary the proposed multi-scale scheme in Fig. 2, where 90% samples in the test image *Peppers* are corrupted. We use the subscript l to indicate the level in the pyramid of downsampled images. The finest level (the original image) is indicated by $l = 0$. The larger is l , the coarser is the downsampled image. We denote the highest level to be $l = L$. First, the level- l image I_l passes a low-pass filter F , which is implemented in our method by averaging the existing pixels in a 2×2 neighborhood on higher resolution. Then, the filter image is downsampled by 2 to get a coarser image I_{l+1} .

$$\mathbf{I}_{l+1} = F(\mathbf{I}_l) \downarrow 2, l = 0, \dots, L - 1.$$

(20)

In this way, we can construct a Laplacian pyramid. In the practical implementation, we construct a tree-level Laplacian pyramid.

CONCLUSION

In this paper, we present an effective and efficient image impulse noise removal algorithm based on hybrid graph Laplacian regularized regression. We utilize the input space and the mapped high-dimensional feature space as two complementary views to address such an ill-posed inverse problem. The framework we explored is a multi-scale Laplacian pyramid, where the intra-scale relationship can be modeled with the implicit kernel graph Laplacian regularization model in input space, while the inter-scale dependency can be learned and propagated with the explicit kernel extension model in mapped feature space. In this way, both local and nonlocal regularity constraints are exploited to improve the accuracy of noisy image recovery.

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